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4. Water Management Optimization Problems

Application of Classic Optimization Tasks in Water Resources Management

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Strengthening of master curricula in water resources management for the Western Balkans HEIs and stakeholders

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1. Optimization

- **Optimization** is an activity, which has an aim to determine *the best possible solution*, at a given criterion and under given circumstances (conditions)
- The best possible solution is the *optimal solution* or *optimum*
- The criterion is called *Objective Function* (OF) – a mathematical expression which shows if the optimum is achieved
  - According to the meaning of the OF the *optimum* is an extremum of the function – it is either **minimum**, or **maximum** of OF.
- The circumstances or conditions under which the optimum is found are called *constraints*.
- Thus, optimization means finding the minimum or maximum of the OF under given set of constraints.

1. Optimization

- Mathematical point of view
  - Optimization can be done in each case, when the number of unknown variables $\boldsymbol{n}$ is greater than the number of equations (or inequalities) $\boldsymbol{m}$ which set interrelation between n variables.
- For example, let's have:
  - n – number of variables;
  - m – number of equations, which link these n variables.
  - When $n < m$, then there is no solution, or only one solution
  - When $n = m$, then there is only one possible solution
    - ✓ We cannot say if the solution is good or bad – it is only one
  - When $n > m$, then there are infinite number of solutions
    - ✓ In that case we may look for the best solution, according to some criteria

1. Optimization

- When $n > m$, then:
  - The equations (or inequalities) which set the interrelation between variables are *the constraints*.
  - In order to find the best possible solution, i.e. optimum solution, we have to specify the OF
  - The OF has to be the function of the variables.
  - There are three kinds of variables:
    - ✓ **decision variables** (non-basic) variables – their number is $\boldsymbol{k = n - m}$
    - ✓ **basic variables** – their number is m
      - These are the variables which can be expressed as function of decision variables by means of the available m equations or inequalities.
    - ✓ **slack variables** – additionally introduced variables, which have the aim to convert inequalities to equations
      - e.g. inequality $a.X_1 + b.X_2 \leq D$ turns to: $a.X_1 + b.X_2 - t_1 = D$, as $t_1 \geq 0$

1. Optimization

- The optimization problem is defined as:

  Find the min/max of the OF

  $$Z = f(X_i),$$

  subject to:

  $$\sum_i a_{ij} X_i \begin{matrix} \leq \\ \geq \end{matrix} b_j$$

  where a_{ij} are the multiplication coefficient of i-th variable in j-th constraint.

  b_j – is the j-th constraint value

- When *all equations* (and inequalities) *are linear* in respect to variables X_i, and also the *OF is a linear function* of variables X_i, then we speak about linear optimization or **linear programming**

1. Optimization

- The following requirements have to be fulfilled, in order to have optimization problem:
  - **Subject to optimization**
    - ✓ In Water Resources Management (WRM) this is Water Management System (WMS) – water supply, irrigation or hydro-power system
  - **Manageability of the subject**
    - ✓ In WRM the WMS, as well as the river runoff are manageable
  - **Optimization criteria**
    - ✓ This is the Objective Function
  - **Optimization method**
    - ✓ There are different methods – analytical, numerical, graphical or experimental
      - the analytical and experimental methods are not suitable for WRM

1. Optimization

- The following types of optimizations are present:
  - ➢ **Static optimization** – when the system subject to optimization is considered (and analyzed) in a static (steady) state.
  - ➢ Dynamic optimization (**Dynamic Programming**)
    - ✓ when (some of) variables in the OF are time-dependant, i.e. they change over time, or
    - ✓ when it is necessary to analyze the systems in several steady states over time.
  - ➢ Linear optimization (**Linear Programming - *LP***) – when all constraints and the OF are in linear relation with variables
  - ➢ Non-linear optimization (**Non-linear Programming**) – when some or all of the constraints and/or OF are in non-linear relation with variables.

1. Optimization

- **Water Resources Management** is actually an optimization problem

The water resources management aims determination of the optimal use of water resources, in accordance with assumed optimization criterion, under set of constraints

- *Subject* to optimizations are water management systems
- *Constraints* are different limitations regarding the size of the systems, the availability of water resources, etc.
- Optimization criterion can be economic, technical and economic, etc.
- *Use of water resources has to be determined.*

2. Linear Programming (LP)

- The optimization problem is defined as:

For the OF: $Z = C_0 + C_1X_1 + C_2X_2 + \ldots + C_nX_n$ find min (or max),

Subject to (constraints):

$$\begin{vmatrix} a_{11}X_1 + a_{12}X_2 + \ldots + a_{1n}X_n \geq b_1 \\ a_{21}X_1 + a_{22}X_2 + \ldots + a_{2n}X_n \leq b_2 \\ \ldots \\ a_{m1}X_1 + a_{m2}X_2 + \ldots + a_{m,n}X_n \geq b_m \end{vmatrix}$$

There are additional constraints in lots of problems – all variables should be non-negative

$$X_i \geq 0$$

- In Linear Programming the min/max of the OF is conditional!
  - ➢ it depends of the constraints!

2. Linear Programming (LP)

- Graphical Interpretation of LP Problems
- When the number of decision variables is $k = 2$, then the solution can be presented in a 2-dimensional space, i.e. in a plane.
  - Let $k = n - m = 2$, and the decision variables are X_1 and X_2.
  - Then all m basic variables can be expressed as function of decision variables X_1 and X_2.

$$
\left| \begin{array}{l}
X_3 = \alpha_{31} X_1 + \alpha_{32} X_2 + \beta_3 \\
X_4 = \alpha_{41} X_1 + \alpha_{42} X_2 + \beta_4 \\
\ldots \\
X_m = \alpha_{m1} X_1 + \alpha_{m2} X_2 + \beta_m
\end{array} \right.
$$

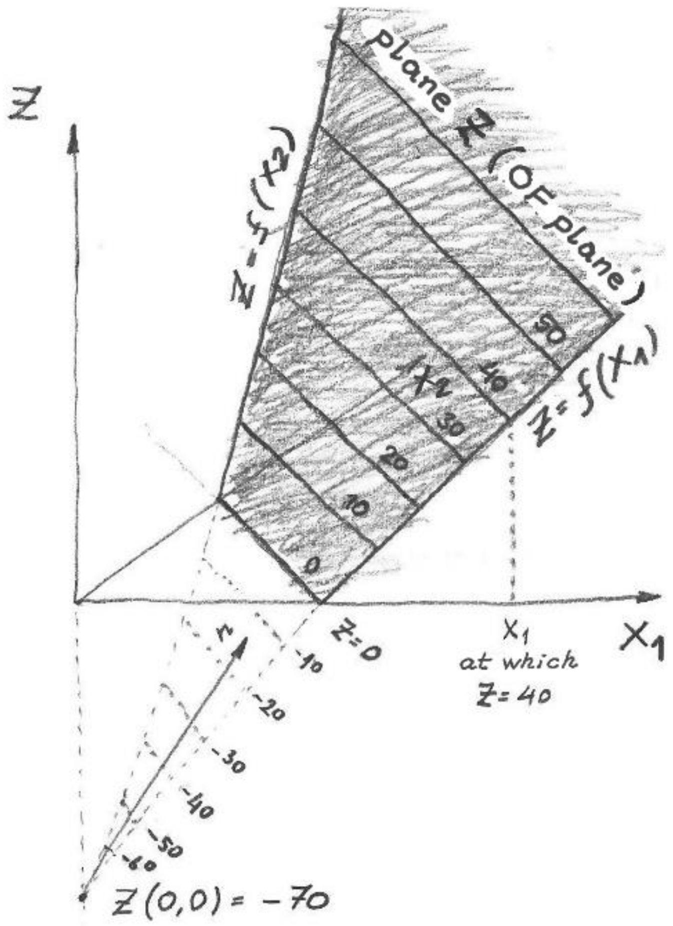
  - The OF is also expressed as function of X_1 and X_2

$$Z = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2$$

2. Linear Programming (LP)

- Graphical Interpretation of LP Problems
- For thar case, the equation of the OF is the equation of the plane in the coordinate system X_1X_2Z

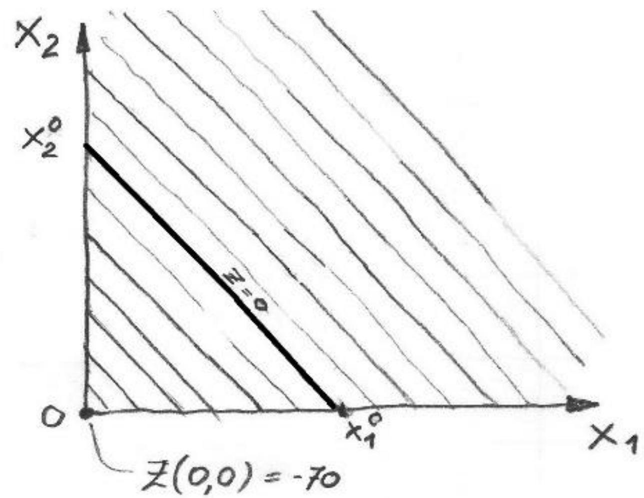
$$Z = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2$$

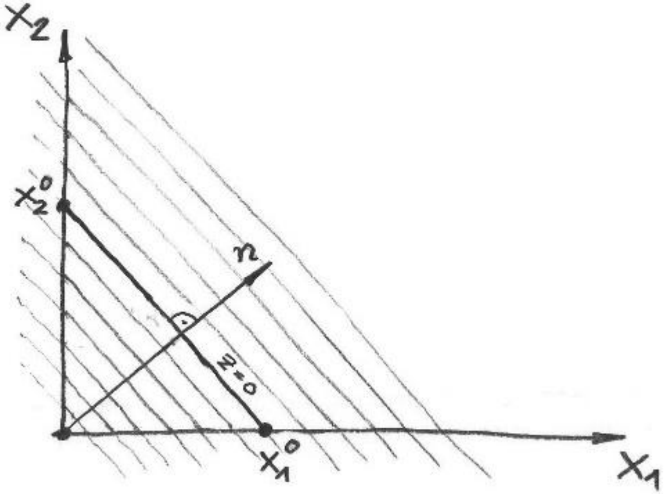


➢ The position in space $X_1 X_2 Z$ depends on:
  - ✓ the presence or absence and on the value of the coefficient γ_0,
  - ✓ the coefficients γ_1 and γ_2 which show the slopes towards the axes X_1 and X_2 – the bigger are the coefficients γ_1 and γ_2 , the steeper are the slopes.

2. Linear Programming (LP)

- Graphical Interpretation of LP Problems
  - ➢ If we consider only 2D space, the OF can be presented in the plane $X_1 0 X_2$ similarly to terrain shape on a topographic (contour) map.





  - ➢ The constant values of OF can be presented as straight contours
    - ✓ On the left picture above the thick line is intersection between OF plane and $X_1 0 X_2$ plane
  - ➢ The biggest slope of OF plane towards $X_1 0 X_2$ plane is in direction of a line n (see right picture above)

2. Linear Programming (LP)

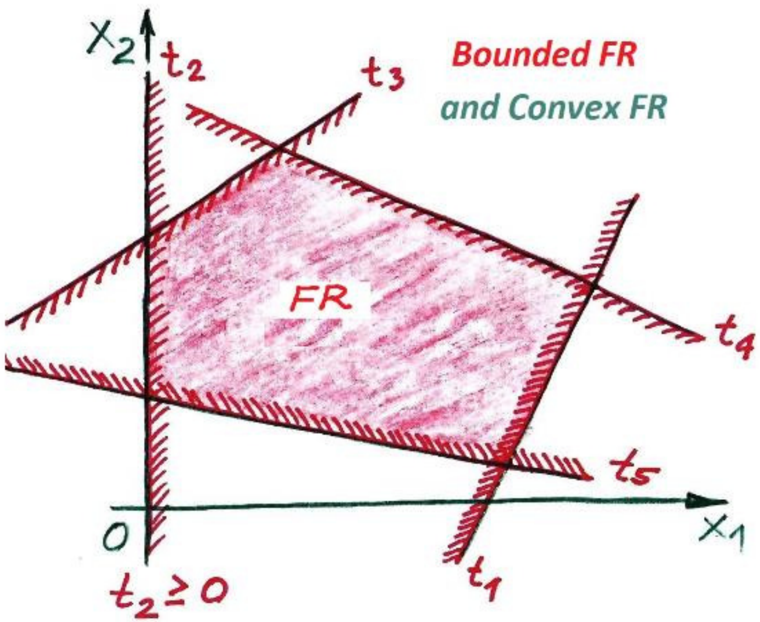
- Graphical Interpretation of LP Problems
  - Each constraint actually represent the equation of a plane, which crosses the $X_1 0 X_2$ plane

$$
\left|
\begin{array}{l}
X_3 = \alpha_{31} X_1 + \alpha_{32} X_2 + \beta_3 \\
X_4 = \alpha_{41} X_1 + \alpha_{42} X_2 + \beta_4 \\
\ldots \\
X_m = \alpha_{m1} X_1 + \alpha_{m2} X_2 + \beta_m
\end{array}
\right.
$$

  - If the constraint is of type *equation*, by substituting any basic variable $X_i = 0$, we will get as result the equation of a line in $X_1 0 X_2$ plane.
    - ✓ All the points along the line are points, which mark *feasible solution*

2. Linear Programming (LP)

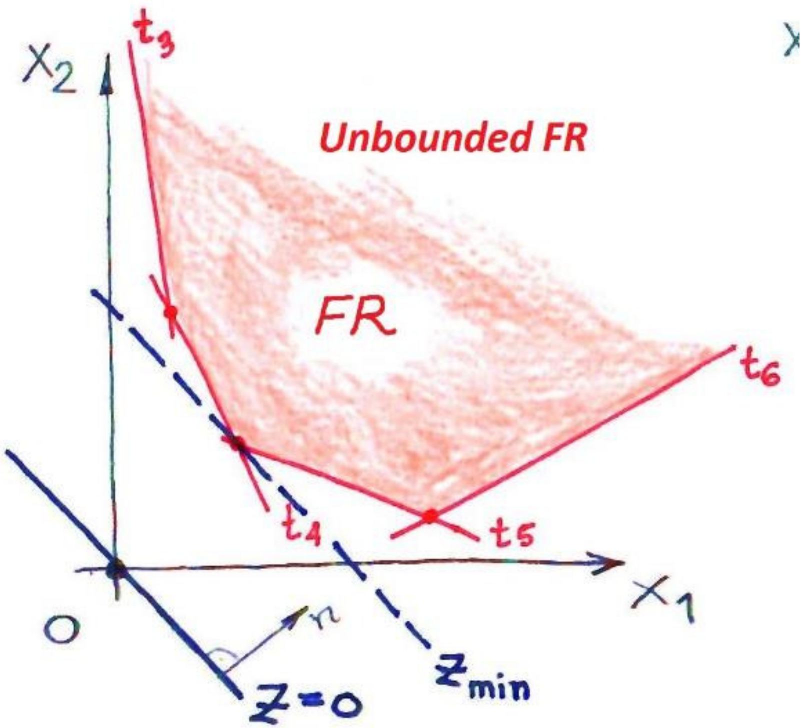
- Graphical Interpretation of LP Problems
  - If the constraint is of type *inequality*, by substituting any basic variable $X_i = 0$, and if we assume that this is equation, we will get as a result the equation of the line in $X_1 0 X_2$ plane.
    - ✓ Because of the inequality, this line splits $X_1 0 X_2$ plane in tow half-planes – one is an area with feasible solutions, and the other one – with non-feasible solutions



- ✓ All constraints create an area in the plane $X_1 0 X_2$
- ✓ If all the constraints are *equations*, then along all lines we have points, which show feasible solutions.
- ✓ If all the constraints are *inequalities*, then the region, which satisfies all inequalities is so-called ***feasible region***, or ***feasible set***.

2. Linear Programming (LP)

- Graphical Interpretation of LP Problems
  - Sometimes, because of number and type of inequalities, the feasible region may not be a compact figure, but it may consist a part of the plane $X_1 0 X_2$.



- It that case it is said that the region is ***unbounded***
- If *feasible region* is *unbounded*, it is important to have the borders of this region facing the origin of the coordinate system $X_1 0 X_2$, if we look for minimum of the OF.
  - ✓ This also means that the origin of the coordinate system is outside of the feasible region

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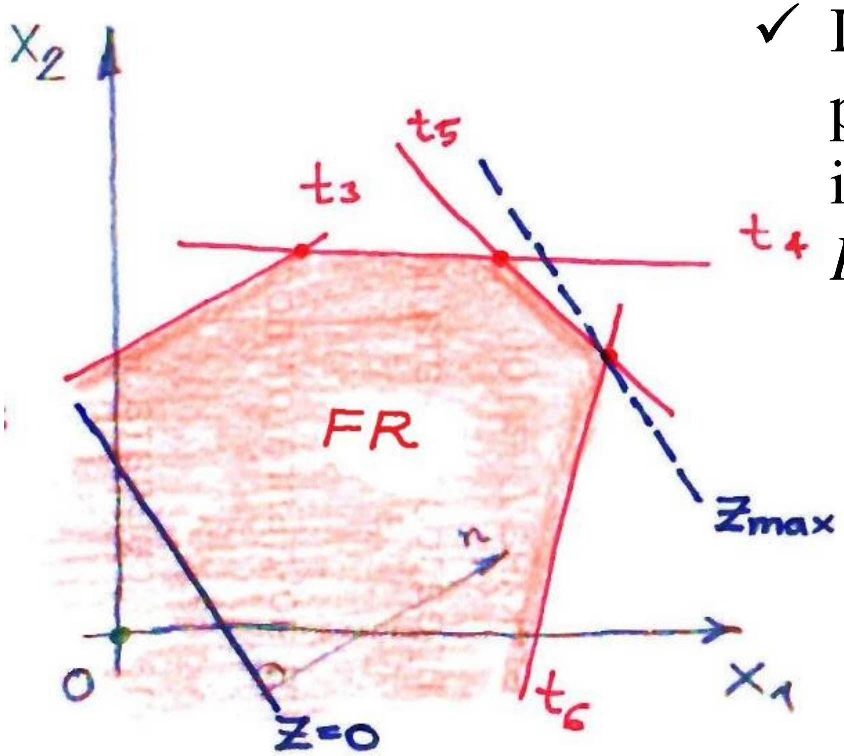
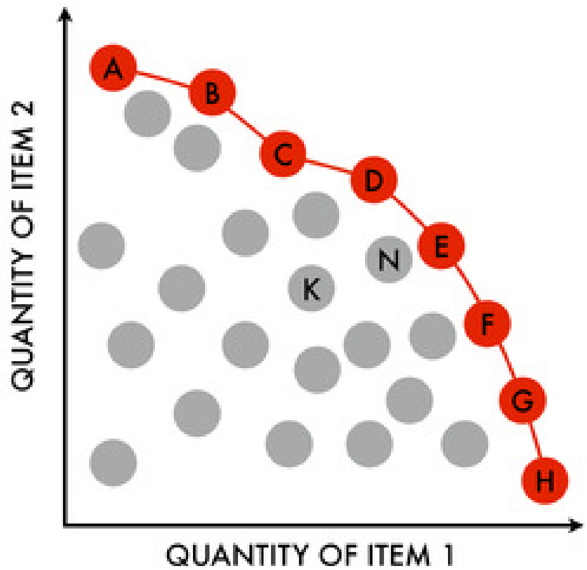
2. Linear Programming (LP)

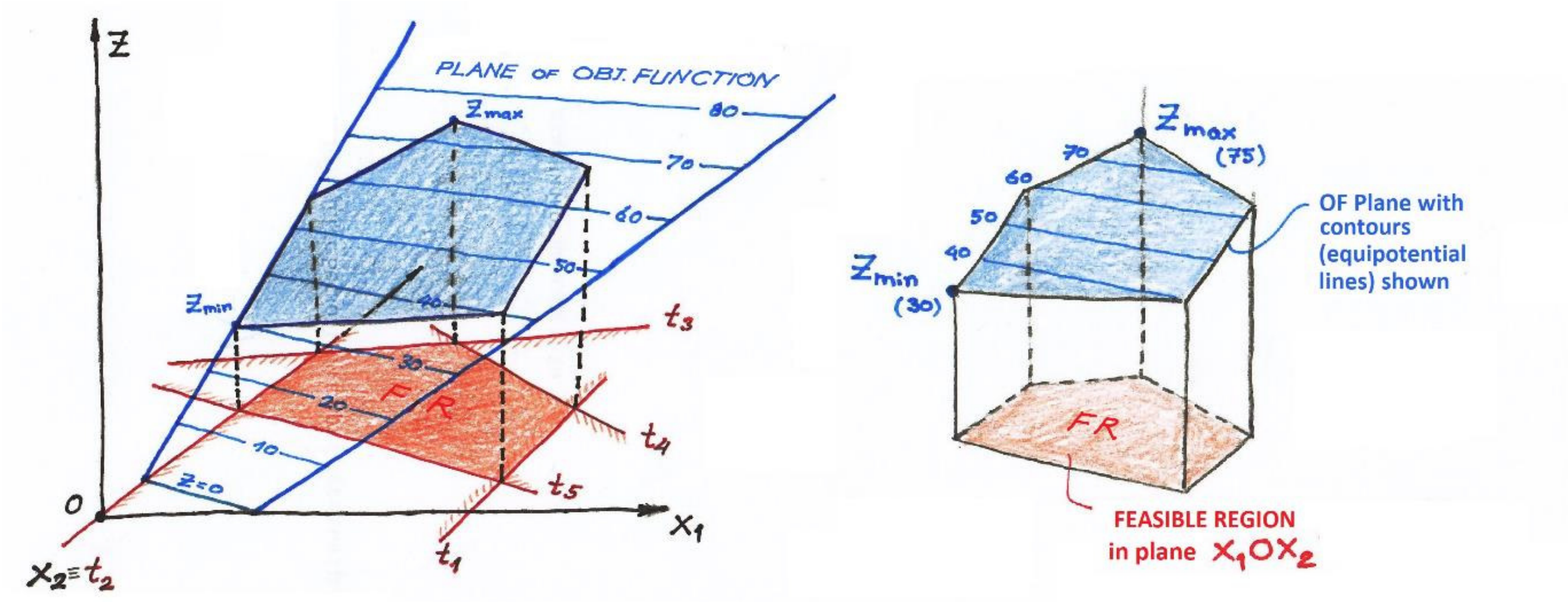
- Graphical Interpretation of LP Problems
  - If we look for maximum of the OF, it is important to have the feasible region restricted from Northeast, i.e. the origin of the coordinate system $X_1 0 X_2$ to be in the feasible solution region.
    - ✓ In case of unbounded feasible region, the polyline which marks the border of the region is known as *Pareto front*, *Pareto frontier* or *Pareto set*.





2. Linear Programming (LP)

- Graphical Interpretation of LP Problems
  - ➢ If the problem is presented in 3D space, then the following picture can be drawn



✓ If the corners of the feasible region are projected towards the plane of the OF, we will get a polygon shape.

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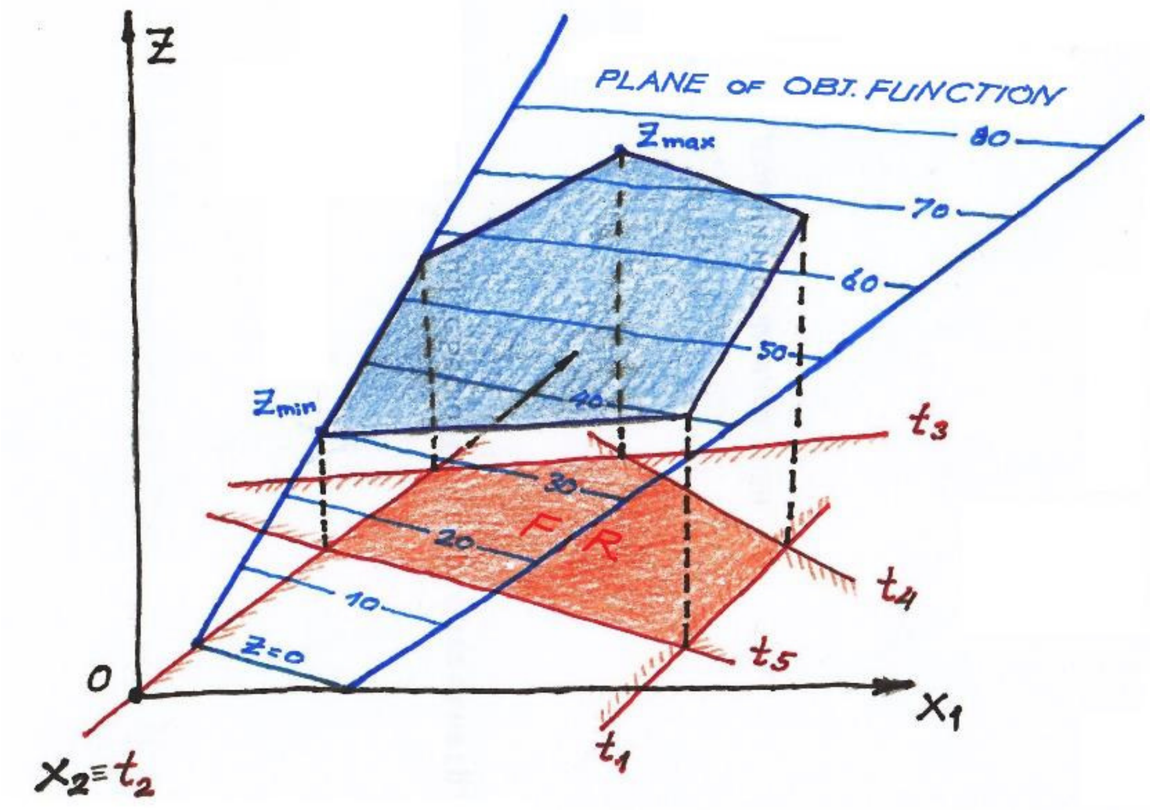
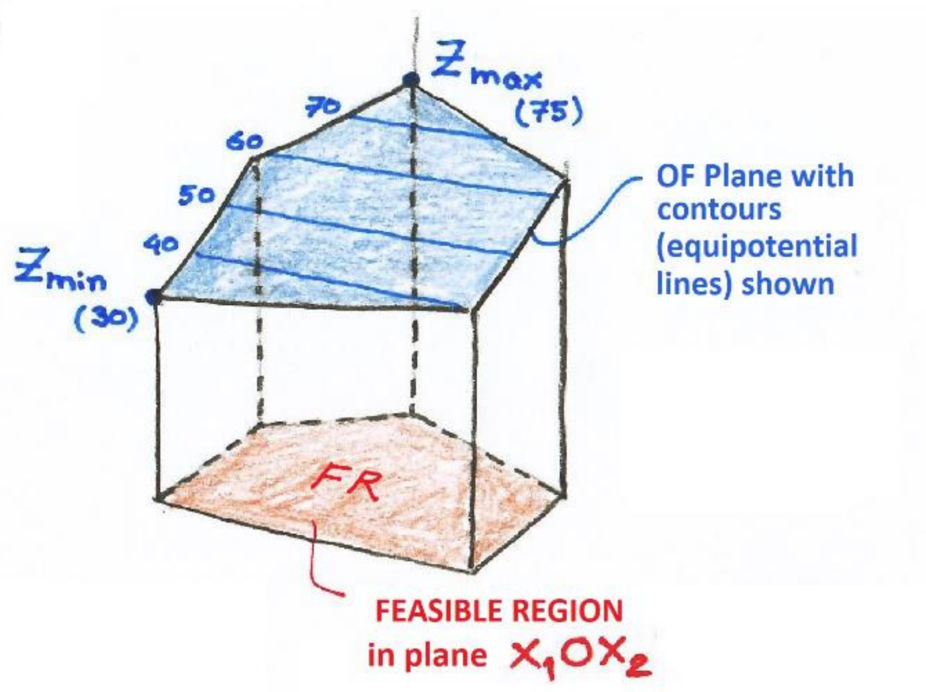
2. Linear Programming (LP)

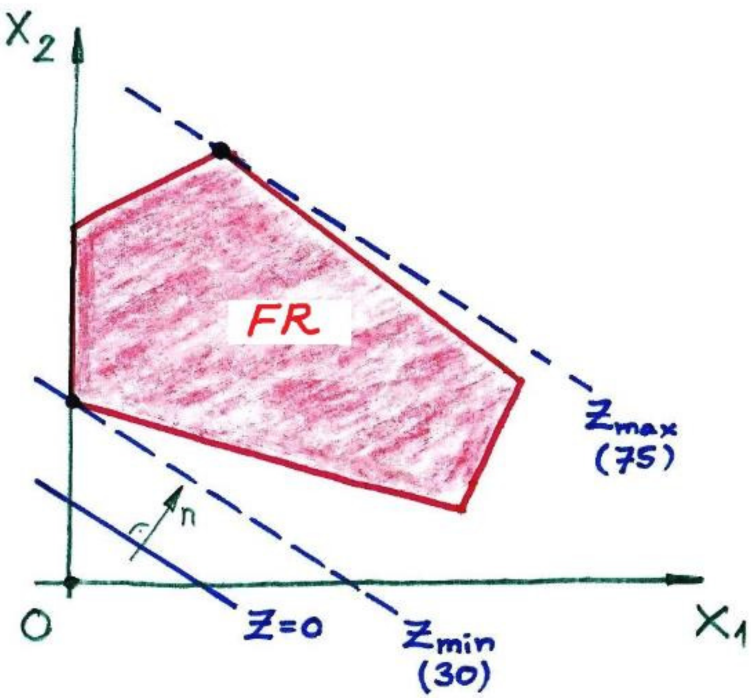
- Graphical Interpretation of LP Problems
  - ➢ It is seen that the maximum or minimum of the OF is found at one of the polygon corners in the plane of the OF





2. Linear Programming (LP)

- Graphical Interpretation of LP Problems
  - In case of a 2D representation of the problem, all lines are drawn in the plane $X_1 0 X_2$.
  - The OF is presented also with lines – projection of equipotential lines of the OF over the plane $X_1 0 X_2$.



- ✓ The extremum (min or max) can be found if we move the line of the OF at value $Z = 0$ (this is intersection between plane $X_1 0 X_2$ and the plane of the OF) in normal direction towards the feasible region.
- ✓ The minimum or maximum ***always*** will be at some of the corners of the feasible region.

2. Linear Programming (LP)

- There are 3 major types of LP problems
  - ➢ **Transportation Problem**
    - ✓ formulated in 1941 by L. Kantorovich (USSR) and F.L. Hitchcock (USA) independently one form another
    - ✓ During the WWII the transportation costs have to be minimized
  - ➢ **Diet Problem or Nutrition Problem**
    - ✓ also known as Mixing or Blending Problem
    - ✓ formulated in 1945 by G. Stigler (Great Britain)
  - ➢ **(Resource) Allocation Problem**
    - ✓ also known as Assignment Problem
    - ✓ formulated in 1947 G.B. Dantzig (USA)
  - ➢ Only Allocation and Transportation Problems are discussed
    - ✓ They have application in Water Resources Management

3. Allocation Problem

- The Allocation problem can be formulated using the following **example**
  - ➢ An agricultural farm has an area of F = 50 ha
  - ➢ Farmer is going to grow two crops - X_1 and X_2.
    - ✓ The profit from crop X_1 (e.g. Maize) is 300 €/ha
    - ✓ The profit from crop X_2 (e.g. Sunflower) is 200 €/ha
  - ➢ The fertilizers needed to obtain these profits are:
    - ✓ for crop X_1 (e.g. Maize) - 500 kg/ha
    - ✓ for crop X_2 (e.g. Sunflower) - 250 kg/ha
  - ➢ According to National requirements the average fertilizer rate per hectare cannot exceed 400 kg/ha, i.e. for an area of F = 50 ha the total amount of fertilizers for the farm is 50.400 = 20 000 kg/year.
  - ➢ Find areas X_1 and X_2 so to maximize farmer's profit.

3. Allocation Problem

- Mathematical formulation:
  - OF: $Z = 300X_1 + 200X_2 \rightarrow \max$
  - Constraints:
    - ✓ $X_1 + X_2 \leq 50$ (area constraint)
    - ✓ $500X_1 + 250X_2 \leq 20000$ (fertilizer constraint)
  - The second constrain maybe reduced to:
    - ✓ $2X_1 + X_2 \leq 80$ (fertilizer constraint)

  - Note that the problem is in so-called ***canonical form***
    - ✓ The canonical form is found when the *maximum* of OF has to be obtained and all constraints are of type "*less or equal*"
    - ✓ Also, canonical form is present in case when the *minimum* of OF has to be obtained and all constraints are of type "*greater or equal*"

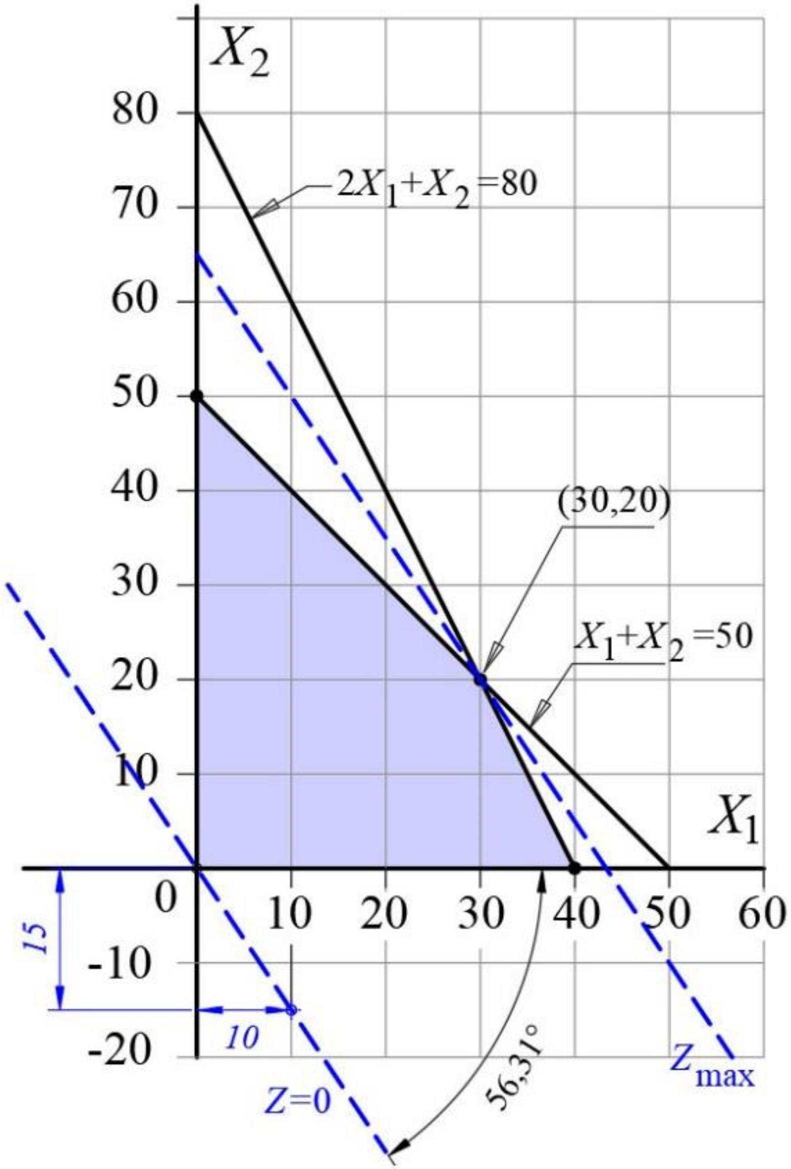
3. Allocation Problem

- Graphical Solution:
  - At first, the two constraints are presented as equations

    $$X_1 + X_2 = 50 \text{ (area constraint)}$$

    $$2X_1 + X_2 = 80 \text{ (fertilizer constraint)}$$

  - The equations represent lines in plane $X_1 0 X_2$
  - They can be drawn if two points of each line are found.
  - We can substitute consecutively X_1=0 and then X_2 =0, to find these points

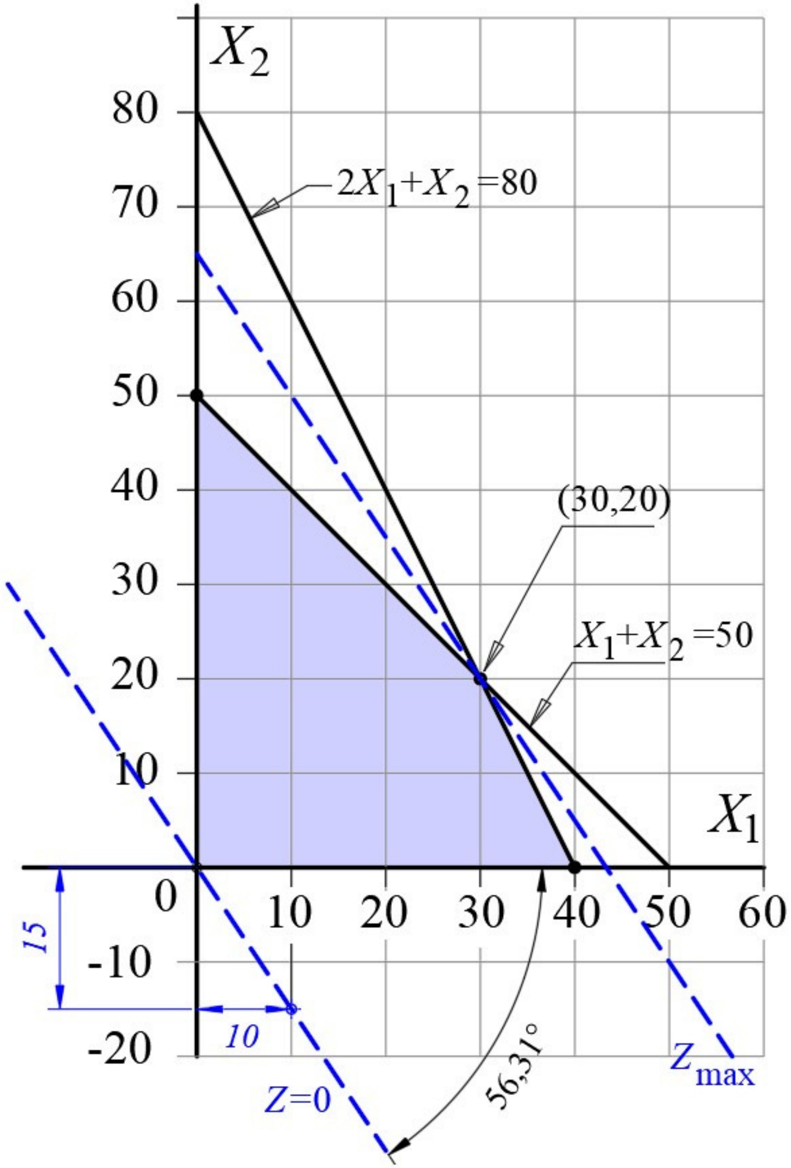


3. Allocation Problem

- Graphical Solution:
  - Second, because the constraints are actually inequalities, the half-planes containing feasible solutions have to be determined

$$X_1 + X_2 \leq 50 \text{ (area constraint)}$$
$$2X_1 + X_2 \leq 80 \text{ (fertilizer constraint)}$$

    - ✓ As it is seen from the inequalities of the two constraints, the origin of the coordinate system $X_1 0 X_2$ (point 0,0) is in the half-plane which satisfies these inequalities, thus the origin (0,0) shows the feasible half-plane.
  - Feasible region is found

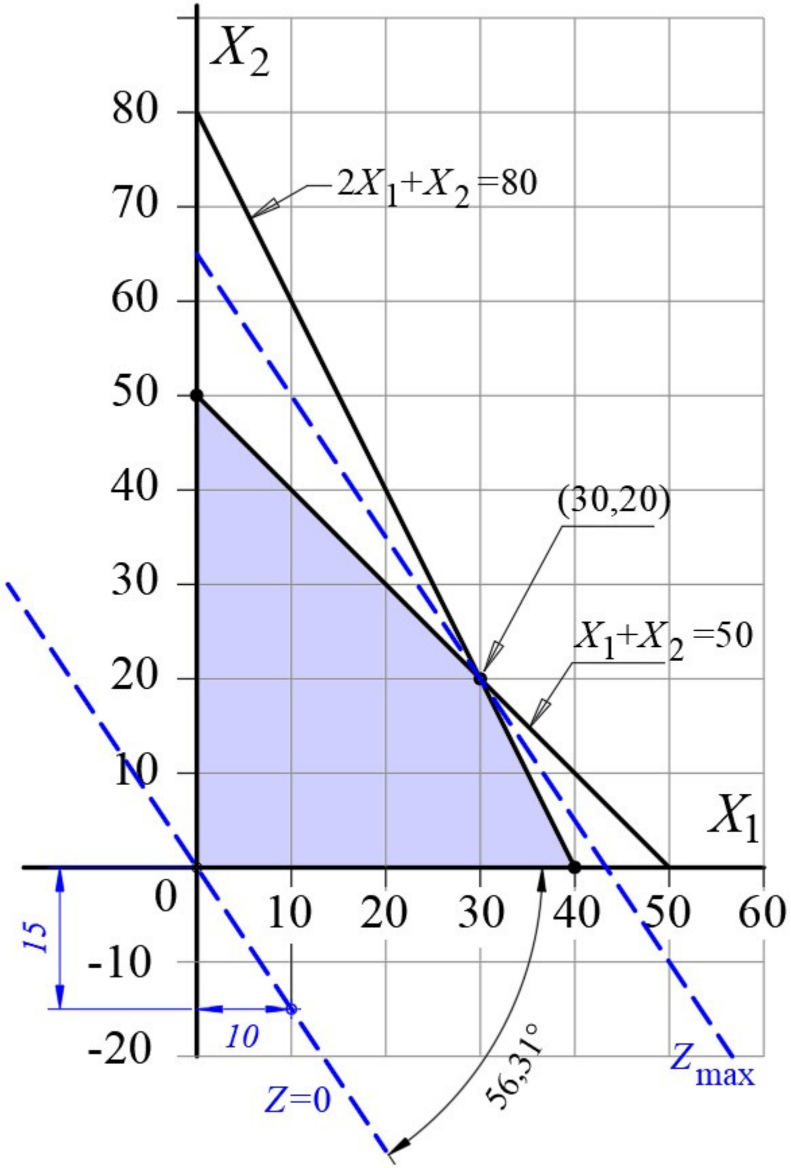


3. Allocation Problem

- Graphical Solution:
  - The line of the OF is drawn
  - The equation is

    $Z = 300X_1 + 200X_2$
  - If $Z = 0$, then the intersection line between the plane $X_1 0 X_2$ and the plane of the OF is found
  - Obviously, when $Z = 0$, the line of the OF intersects the origin 0,0.
  - The position of OF line in the plane $X_1 0 X_2$ is found by rearranging the equation $300X_1 + 200X_2 = 0$

$$\frac{X_2}{X_1} = -\frac{300}{200}$$



3. Allocation Problem

- Graphical Solution:
  - It is known that:

$$tg\alpha = \frac{X_2}{X_1}$$

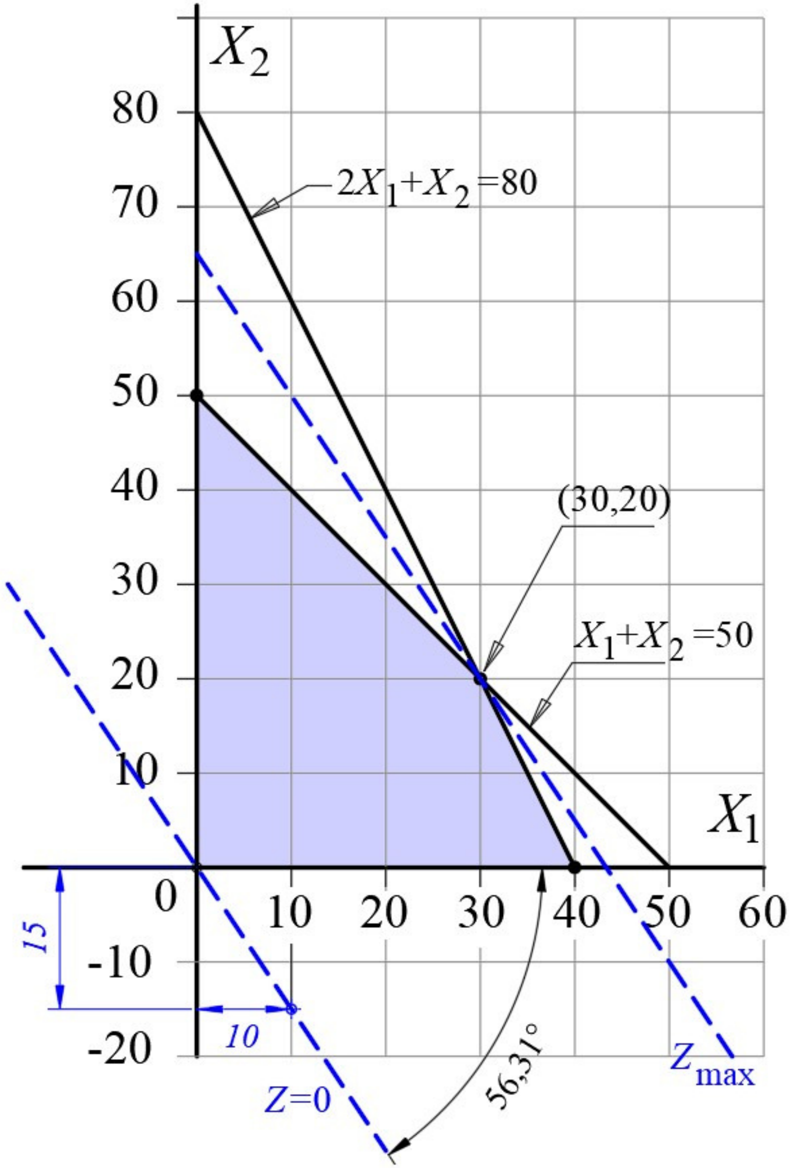
where α is the angle between the axis X_1 and the line of the OF

  - Thus we obtain:

$$tg\alpha = \frac{X_2}{X_1} = -\frac{300}{200} = -1,5$$

or $\alpha = -56,31°$

  - The position of the OF line when $Z = 0$ is found (see figure)



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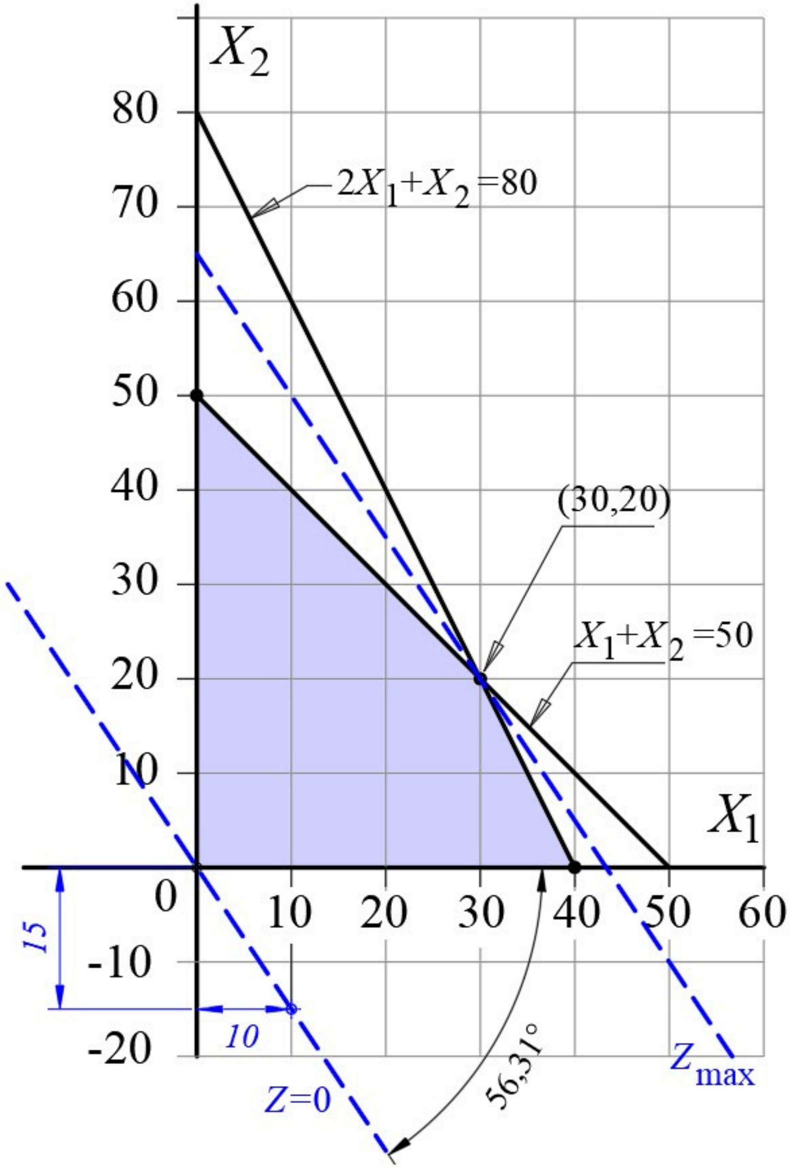
3. Allocation Problem

- Graphical Solution:
  - The optimal solution is found by translating the OF line in normal direction till it intersects that corner at which the line do not crosses the feasible region.
    - ✓ For the current example, this is the **point (30,20)**
  - Replacing $X_1 = 30$ and $X_2 = 10$ in the OF equation, we find:
  - $Z_{max} = 300.30 + 200.20 = 13000$ €.

N.B. One can try to estimate the OF value for any other point in the feasible region to see if the result is correct



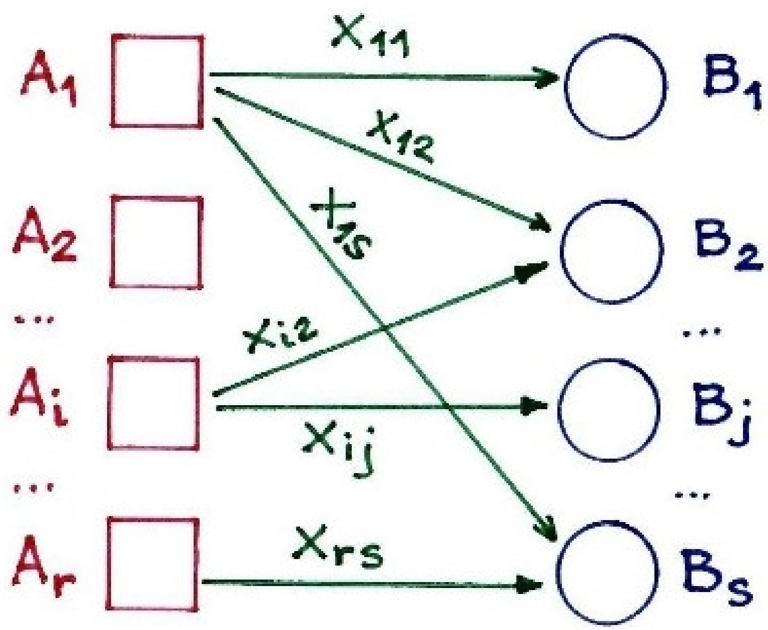
3. Allocation Problem

- Allocation problem in Water Resources Management
- Example
  - A reservoir is planned with a live storage V_{live} = 20 mil. m^3.
  - The reservoir can supply water for an Irrigation system and an Industrial complex, which will contribute to National Economy:
    - ✓ profit 0,1 €/m^3 from the Irrigation system;
    - ✓ profit 0,3 €/m^3 from the Industrial complex.
  - The energy consumption for delivery the water to users is:
    - ✓ to Irrigation System – 0,05 kWh/m^3.
    - ✓ to Industrial complex – 0,20 kWh/m^3.
  - The available energy for the Irrigation System and Industrial complex is estimated to 2,4 mil. kWh.
  - Find volumes of water to be supplied to two users.

4. Transportation Problem

- Definition of Transportation Problem
  - ➢ There are number of r producers A_i and number of s stores B_j.
  - ➢ Each producer A_i has it own capacity $W^A{}_i$ to supply given product to stores.
  - ➢ Each store A_i has its own capacity $W^B{}_j$ to receive the products.
  - ➢ The transported number of products from producer A_i to store B_j is X_{ij}.
  - ➢ The unit cost for transportation from producer A_i to store B_j is C_{ij}.

N.B. It is assumed that all products will be sold from stores B_j.



4. Transportation Problem

- Definition of Transportation Problem
  - The task is to find the *minimum cost* for transportation from producers A_i to stores B_j, i.e.
  - the **OF** is:

$$Z = X_{11}C_{11} + X_{12}C_{12} + X_{1s}C_{1s} + \ldots + X_{21}C_{21} + X_{22}C_{22} + \ldots + X_{r1}C_{r1} + \ldots + X_{rs}C_{rs},$$

or

$$Z = \sum_i \sum_j X_{ij}C_{ij}, \text{ where } i = 1 \div r; \quad j = 1 \div s.$$

  - Subject to constraints:

$$\sum_j X_{ij} \le W_i^A$$ - the capacity of each producer should not be exceeded

$$\sum_i X_{ij} \le W_j^B$$ - the capacity of each store should not be exceeded

N.B. It is typical for the problem to have constraints as equations, not as inequalities

4. Transportation Problem

- Types of Transportation Problem
  - **The problem is *balanced***
    - ✓ When the total production (supply) is equal to total demand:

$$\sum_i W_i^A = \sum_j W_j^B$$

  - **The problem is *unbalanced***
    - ✓ When the total production (supply) is not equal to total demand
    - ✓ It is possible to have a ***Surplus*** or

$$\sum_i W_i^A > \sum_j W_j^B$$

    - ✓ to have a ***Deficit***:

$$\sum_i W_i^A < \sum_j W_j^B$$

4. Transportation Problem

- Types of Transportation Problem
  - ➢ It is difficult, if not impossible to solve *unbalanced* problem
  - ➢ To make the problem *balanced*, it is introduced:
    - ✓ A ***dummy* store** (or *dump*), in case of a **surplus**
      - – The capacity of a dummy store is $W_{s+1}^{B} = \sum_{i} W_{i}^{A} - \sum_{j} W_{j}^{B}$
    - ✓ A dummy producer, in case of a deficit
      - – The capacity of a dummy producer is $W_{r+1}^{A} = \sum_{j} W_{j}^{B} - \sum_{i} W_{i}^{A}$
    - ✓ In general, the case of deficit is not typical for Transportation problem.
  - ➢ The unit costs for transportation *to* dummy store $C_{i,s+1}$ or *from* dummy producer $C_{r+1,j}$ is assumed ***enormously high***, compared to other unit costs.

4. Transportation Problem

- Objective Function
  - The OF $Z = \sum_i \sum_j X_{ij} C_{ij}$

is actually a product of two matrices:

Matrix of Quantities and Matrix of Unit Costs

	B_1	B_2	…	B_j	…	B_s
A_1	X_{11}	X_{12}		X_{1j}		X_{1s}
A_2	X_{21}	X_{22}		X_{2j}		X_{2s}
…						
A_i	X_{i1}	X_{i2}		X_{ij}		X_{is}
…						
A_r	X_{r1}	X_{r2}		X_{rj}		X_{rs}

	B_1	B_2	…	B_j	…	B_s
A_1	C_{11}	C_{12}		C_{1j}		C_{1s}
A_2	C_{21}	C_{22}		C_{2j}		C_{2s}
…						
A_i	C_{i1}	C_{i2}		C_{ij}		C_{is}
…						
A_r	C_{r1}	C_{r2}		C_{rj}		C_{rs}

✓ Usually the Unit Costs are constants, regardless of the quantities transported.

4. Transportation Problem

- The constraints
  - The constraints have to be converted to equations, if such exist
  - Constraints, regarding production (supply) will be

$$\left|\begin{array}{l} X_{11}+X_{12}+\ldots+X_{1s}=W_1^A \\ X_{21}+X_{22}+\ldots+X_{2s}=W_2^A \\ \ldots \\ X_{r1}+X_{r2}+\ldots+X_{rs}=W_r^A \end{array}\right.$$

  - Constraints, regarding stores (demand) will be:

$$\left|\begin{array}{l} X_{11}+X_{21}+\ldots+X_{r1}=W_1^B \\ X_{12}+X_{22}+\ldots+X_{r2}=W_2^B \\ \ldots \\ X_{1s}+X_{2s}+\ldots+X_{rs}=W_s^B \end{array}\right.$$

  - It is understandable, without saying that $X_{ij} \geq 0$.

4. Transportation Problem

- Variables
  - ➢ The number of constraints: $m = r + s - 1$
    - ✓ It is "*minus* 1", because one of the constraints is dependable, due to equality $\sum_i W_i^A = \sum_j W_j^B$
  - ➢ Variables: $n = r \times s$
    - ✓ If not all constraints are equations and the number of inequalities is t, then the number of *slack variables* will be t. In that case, the number of variables will be $n = r \times s + t$
  - ➢ Basic variables: m
  - ➢ Decision variables: $k = n - m = r \times s - (r + s - 1) = r(s-1) - (s-1)$

$$k = (r-1)(s-1)$$

  - ➢ Usually the number k is too big, or at least bigger than 2, to solve the Transportation problem graphically

4. Transportation Problem

- <u>Transportation Problem in Water Resources Management</u>
  - ➢ The Transportation problem can also be found in WRM
  - ➢ Typical example is the task for determination of water delivery from multiple sources to multiple users.
    - ✓ If water sources are A_i and the water users are B_j, then the problem consists of finding the minimum cost for water delivery
  - ➢ This problem can be solved in two different cases:
    - ✓ Project case – during preliminary research for design of water supply network or irrigation network
      - – **Minimum investments in construction of the network is found**
    - ✓ Exploitation case – during exploitation stage of already existing networks, if there are canals/pipelines from each source to several (or all) water users
      - – **Minimum expenses for water delivery in a given period of time is found**

4. Transportation Problem

- Transportation Problem in Water Resources Management
  - ➢ The Transportation problem in WRM has its specifics
  - ➢ For the example of water delivery from multiple sources to multiple users.
    - ✓ In both cases – project and exploitation – the unit costs are NOT constant.
    - ✓ In project case – the canal/pipeline sizes depend on the design discharges – the bigger are the discharges, the higher are the costs
      - **But the relation *Q-Cost* is not linear!**
    - ✓ In exploitation case – the expenses for delivery depend on volumes – the bigger are the volumes, the higher are the expenses.
      - **In case of delivery by open canals – the expenses are almost constant or in approximately linear relation to volumes**
      - **In case of pressurized flow (by pumps) – the expenses are in non-linear relation to volumes**

4. Transportation Problem

- Transportation Problem in Water Resources Management
  - The Transportation problem in WRM can be solved using standard procedure, but taking into account the specifics
    - ✓ The OF is changed: $Z = \sum_i \sum_j Z_{ij}$

    where Z_{ij} are the total costs or total expenses, estimated as function of discharges or volumes
    - ✓ The functions $Z_{ij} - Q$ or $Z_{ij} - V$ have to be established
    - ✓ These functions can be polynomial, e.g.:

$$Z_{ij} = a_0 + a_1 V + a_2 V^2 + a_3 V^3 + a_4 V^4, \text{€}$$

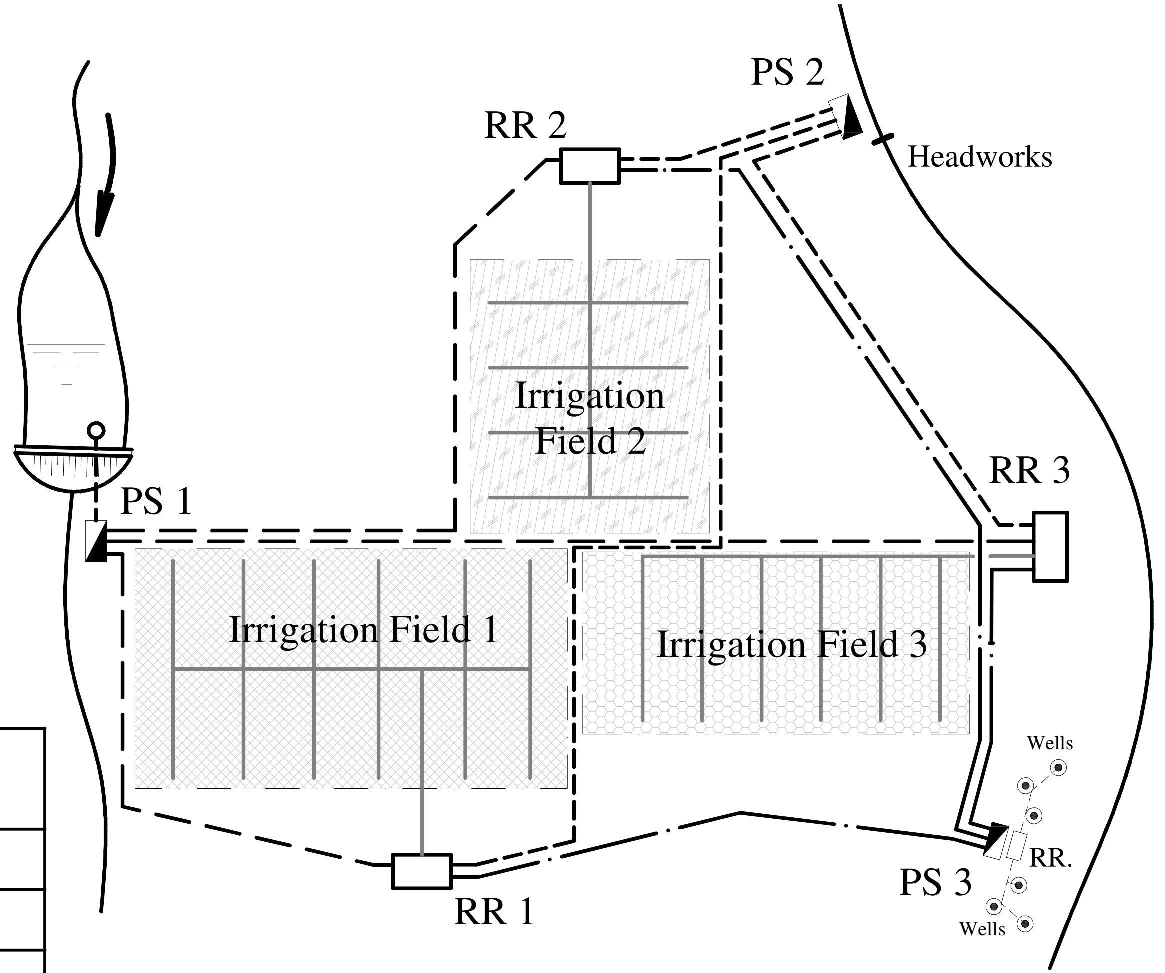
4. Transportation Problem

- Transportation Problem in Water Resources Management
  - Example for student's task

✓ Each water source can deliver water to each water user

✓ It is possible to have limited number of connections between sources and users

✓ It is possible to have balanced or unbalanced task



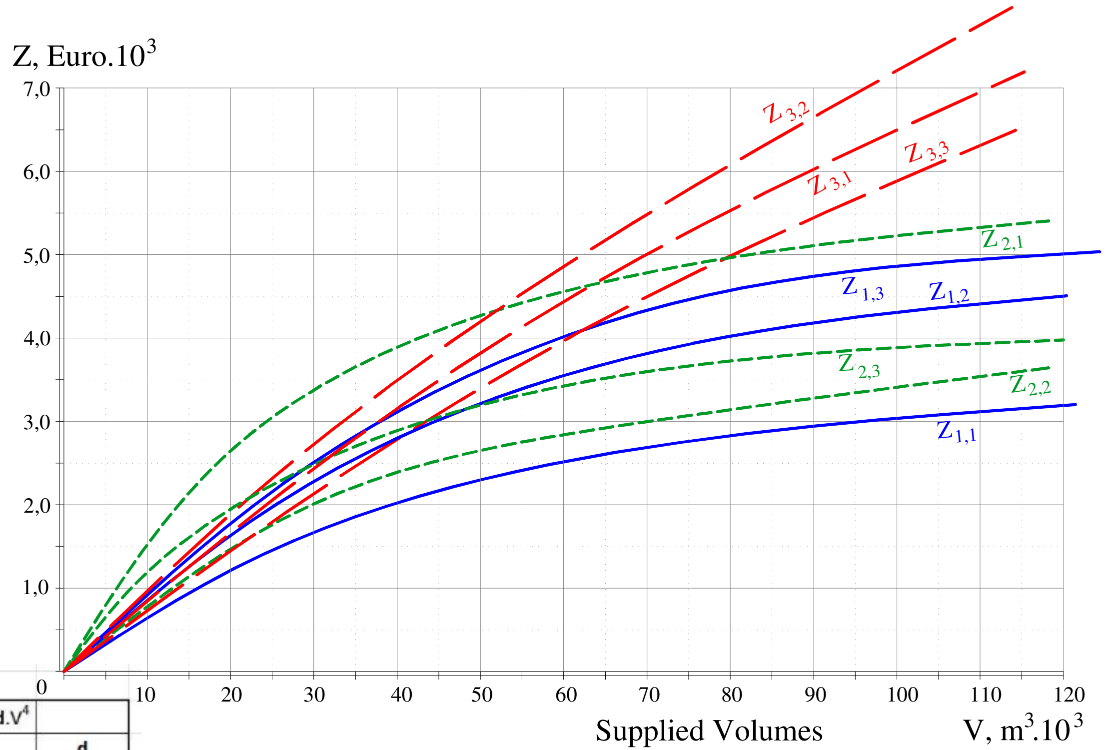
Water Source	Supply Capacity $10^3.m^3$	Water User	Demand, $10^3.m^3$
A_1 (PS 1)	90	B_1 (RR 1)	90
A_2 (PS 2)	70	B_2 (RR 2)	80
A_3 (PS 3)	50	B_3 (RR 3)	70

4. Transportation Problem

- Transportation Problem in Water Resources Management
  - ➢ Example for student's task

✓ The costs for water transportation are not linear

✓ The costs are presented with polynomials



COSTS READ FROM THE GRAPH

	Volumes of Water				
Cost	0	20	40	70	100
Z 1,1	0	1,21	2,02	2,69	3,03
Z 1,2	0	1,63	2,8	3,82	4,31
Z 1,3	0	1,77	3,11	4,33	4,86
Z 2,1	0	2,65	3,89	4,78	5,23
Z 2,2	0	1,46	2,39	3	3,41
Z 2,3	0	1,95	2,89	3,6	3,89
Z 3,1	0	1,66	3,15	5	6,5
Z 3,2	0	1,88	3,49	5,48	7,21
Z 3,3	0	1,44	2,78	4,5	5,88
Z 4,1					
Z 4,2					
Z 4,3					

	Polynomial terms: $Z = a.V+b.V^2+c.V^3+d.V^4$			
Cost	a	b	c	d
Z 1,1	0,0725	-0,0006	0,00000178	
Z 1,2	0,0956	-0,0007	0,00000175	
Z 1,3	0,0985	-0,0005	-0,000002	2E-08
Z 2,1	0,1866	-0,0033	0,00003	-1E-07
Z 2,2	0,0921	-0,001	0,0000042	
Z 2,3	0,1206	-0,0014	0,000006	
Z 3,1	0,0886	-0,0003	0,00000065	
Z 3,2	0,1027	-0,0004	0,0000009	
Z 3,3	0,0751	-0,0001	-0,0000008	
Z 4,1	1	0	0	
Z 4,2	1	0	0	
Z 4,3	1	0	0	

4. Transportation Problem

- <u>Transportation Problem in Water Resources Management</u>
  - ➢ Example for student's task
    - ✓ The optimal solution is found by MS Solver Tool

F19 | =SUM(C19:E19)

MATRIX OF VOLUMES

	B1	B2	B3	Sum	Available V
A1	30	10	0	40	40
A2	0	40	0	40	40
A3	0	10	10	20	20
A4	0	0	20	20	20
Sum	30	60	30		120
Needed V	30	60	30	120	

MATRIX OF COSTS

	B1	B2	B3	Sum	
A1	1,68306	0,88775	0	2,571	
A2	0	2,3528	0	2,353	
A3	0	0,9879	0,7402	1,728	
A4	0	0	20	20,000	
				26,652	
Sum	1,68306	4,22845	20,7402	26,652	Check